Please check the examination details below before entering your candidate information

Candidate surname

Other names

**Pearson Edexcel
Level 3 GCE**

Centre Number

Candidate Number

Paper reference **8FM0/25**

Further Mathematics

**Advanced Subsidiary
Further Mathematics options
25: Further Mechanics 1
(Part of options C, E, H and J)**

You must have: Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need*.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Unless otherwise indicated, whenever a value of g is required, take $g = 9.8\,\mathrm{m\,s^{-2}}$ and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 40. There are 4 questions.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- Good luck with your examination.

Turn over

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**Pearson**

1.
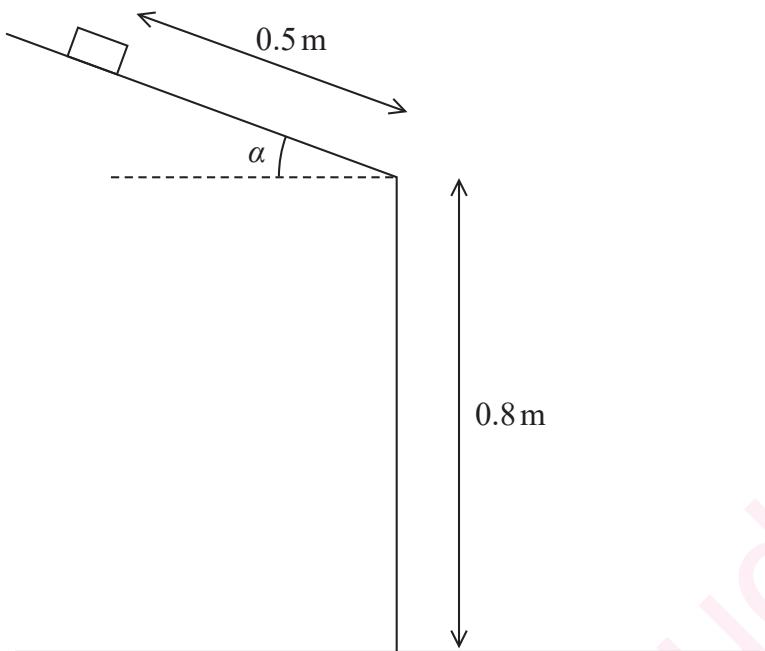
Figure 1

A small book of mass m is held on a rough straight desk lid which is inclined at an

angle α to the horizontal, where $\tan \alpha = \dfrac{3}{4}$. The book is released from rest at a distance

of 0.5 m from the edge of the desk lid, as shown in Figure 1. The book slides down the desk lid and then hits the floor that is 0.8 m below the edge of the desk lid. The coefficient of friction between the book and the desk lid is 0.4 μ

The book is modelled as a particle which, after leaving the desk lid, is assumed to move freely under gravity.

(a) Find, in terms of m and g, the magnitude of the normal reaction on the book as it slides down the desk lid.

(2)

(b) Use the work-energy principle to find the speed of the book as it hits the floor.

(5)

$$\tan \alpha = \frac{3}{4} \qquad \begin{array}{c} \sqrt{3^2+4^2} \\ = 5 \end{array} \qquad \therefore \cos \alpha = \frac{4}{5}$$

$$\sin \alpha = \frac{3}{5}$$

(a) **Diagram**

Use $\Sigma F_y = 0$ to get R

$R = mg \cos \alpha = \frac{4}{5} mg$ **M1**

$\therefore R = \frac{4}{5} mg$ N   normal reaction force **A1**

P 6 6 7 9 3 A 0 2 1 6

Question 1 continued

(b)

⭐ Work-Energy Principle: an <u>increase</u> of KE/GPE is caused by an equal amount of <u>positive</u> work done on the body (e.g. engine) and a <u>decrease</u> of KE/GPE is caused by an equal amount of <u>negative</u> work done on the body (e.g. friction).

★ Remember the work-energy formulae:

Either: WD by force + KE_i + GPE_i = KE_F + GPE_F + WD against friction

workdone · initial kinetic · initial grav. potential · final kinetic · final grav. potential · work lost to friction

OR: WD by force + KE_i + GPE_i − WD by friction = KE_F + GPE_F

workdone · initial kinetic · initial grav. potential · we subtract this since it leaves the system as heat! · final kinetic · final grav. potential

⭐ Formulae for KE and GPE:

$$KE = \frac{1}{2}mv^2 \quad \text{velocity, mass}$$

$$GPE = mgh \quad \text{change in height, mass, } g = 9.8 \, ms^{-2}$$

Diagram
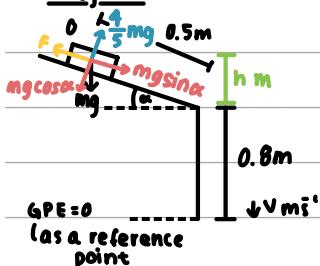
Substitute:

$$\frac{1}{2}m(0) + mg(h + 0.8) - 0.5F = \frac{1}{2}mv^2 + mg(0) \quad \boxed{M1}$$

→ Get h:

$$\sin\alpha = \frac{3}{5} \longrightarrow \sin\alpha = \frac{h}{0.5} \xrightarrow{\text{equate}} \frac{h}{0.5} = \frac{3}{5} \quad \underline{h = 0.3m}$$

→ Get F:

since it's moving, $F_{max} = \mu R$

$$F = 0.4 \times \frac{4}{5}mg = \underline{0.32mg}$$

Substitute:

$$mg(0.3 + 0.8) - 0.5(0.32mg) = \frac{1}{2}mv^2$$

$\boxed{M1 \, M1 \, A1}$ $1.1mg - 0.16mg = \frac{1}{2}mv^2$ cancel m's

$$0.94g = \frac{1}{2}v^2$$

$$1.88g = v^2$$

$$\boxed{v = 4.29 \, ms^{-1} \text{ to 3sf}} \quad \boxed{A1}$$

speed when it hits the floor

Question 1 continued

Question 1 continued

(Total for Question 1 is 7 marks)

2.
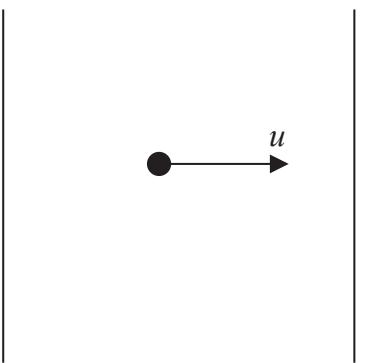
Figure 2

A particle of mass em is at rest on a smooth horizontal plane between two smooth fixed parallel vertical walls, as shown in the plan view in Figure 2. The particle is projected along the plane with speed u towards one of the walls and strikes the wall at right angles. The coefficient of restitution between the particle and each wall is e and air resistance is modelled as being negligible.

Using the model,

(a) find, in terms of m, u and e, an expression for the total loss in the kinetic energy of the particle as a result of the first two impacts.

consider 2 impacts!

(3)

Given that e can vary such that $0 < e < 1$ and using the model,

(b) find the value of e for which the total loss in the kinetic energy of the particle as a result of the first two impacts is a maximum,

(4)

(c) describe the subsequent motion of the particle.

(2)

Question 2 continued

(a) <u>Diagram</u>

Before   After 1st After 2nd
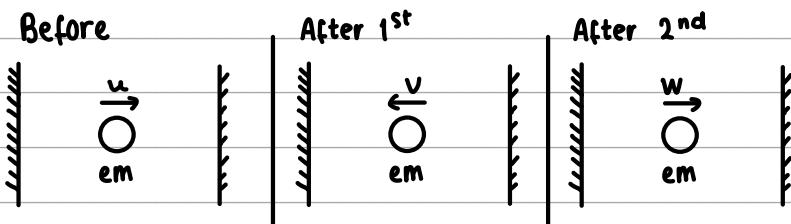
Using NLR to get v:

→ multiply by e and reverse direction by multiplying by -1

$$-eu = v \qquad \text{1}^{st} \text{ collision}$$

$$(-eu) \times -e = w$$

$$w = e^2u \qquad \text{2}^{nd} \text{ collision}$$

Formula for Kinetic Energy: **B1**

$$KE = \frac{1}{2}mv^2$$
$$\underset{mass}{} \quad \underset{velocity}{}$$

Substitute:

$$\Delta KE = KE_I - KE_F$$

$$= \frac{1}{2}(em)\underset{(\text{before})}{(u)^2} - \frac{1}{2}(em)\underset{(\text{after 2}^{nd})}{(e^2u)^2} \quad \textbf{M1}$$

$$= \frac{emu^2}{2} - \frac{em}{2}(e^4u^2)$$

$$= \frac{emu^2}{2} - \frac{e^5mu^2}{2}$$

$$= \frac{1}{2}mu^2(e - e^5) \text{ J lost} \quad \textbf{A1}$$

(b) To get maximum KE lost, let's differentiate our answer to (a) wrt. e

$$\Delta KE = \frac{1}{2}mu^2(e - e^5)$$

$$\frac{d\Delta KE}{de} = \frac{1}{2}mu^2(1 - 5e^4) \quad \textbf{M1A1}$$

and now let's set our $\frac{d\Delta KE}{de} = 0$ to get the value of e for which this occurs:

$$0 = \frac{1}{2}mu^2(1 - 5e^4) \quad \textbf{M1}$$

$$0 = 1 - 5e^4$$

$$5e^4 = 1$$

$$e^4 = \frac{1}{5}$$

$$e = 0.67 \text{ to 2sf} \quad \textbf{A1}$$

(c) The particle continues to hit the walls repeatedly. Since it loses energy during each collision, its speed decreases. **B1B1**

Question 2 continued

Question 2 continued

(Total for Question 2 is 9 marks)

P 6 6 7 9 3 A 0 9 1 6

3. The total mass of a cyclist and his bicycle is 100 kg.

In all circumstances, the magnitude of the resistance to the motion of the cyclist from non-gravitational forces is modelled as being kv^2 N, where $v\,\text{m s}^{-1}$ is the speed of the cyclist.

no force downwards!

The cyclist can freewheel, without pedalling, down a slope that is inclined to the horizontal at an angle α, where $\sin\alpha = \dfrac{1}{35}$, at a constant speed of $V\,\text{m s}^{-1}$

case A

When he is pedalling up a slope that is inclined to the horizontal at an angle β, where $\sin\beta = \dfrac{1}{70}$, and he is moving at the same constant speed $V\,\text{m s}^{-1}$, he is working at a constant rate of P watts.

case B

(a) Find P in terms of V.

(7)

If he pedals and works at a rate of $35V$ watts on a horizontal road, he moves at a constant speed of $U\,\text{m s}^{-1}$

(b) Find U in terms of V.

(4)

(a) Diagram – for Case A
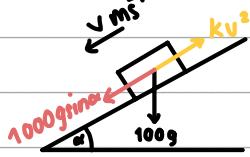
We are told that speed is constant ∴ use $\Sigma F_x = 0$

$100g\sin\alpha = kV^2$ M1A1

$\dfrac{100g}{35} = kV^2$

Diagram – for Case B
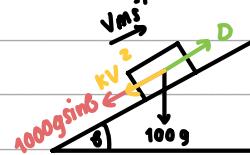
We are told that speed is constant ∴ use $\Sigma F_x = 0$ again:

$100g\sin\beta + kV^2 = D$ M1A1

from case A

$100g \times \dfrac{1}{70} + \dfrac{100g}{35} = P$

$\dfrac{300g}{70} = D$

To get D we will use Power.

Formula for Power:

Power (w) ⌣ $P = Dv$

Driving force (N) Velocity (m s⁻¹)

$P = P$ W $P = Dv \longrightarrow D = \dfrac{P}{V}$ M1

$D = D$ N

$V = v$

Substitute D:

$\dfrac{300g}{70} = \dfrac{P}{V}$ M1

$P = \dfrac{30g}{7}V$

A1 P = 42V to 2sf   P in terms of V

Question 3 continued

(b) Diagram - horizontal road
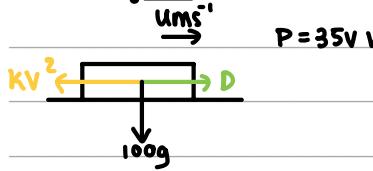
$P = 35v$ W

Since the speed is constant, use $\Sigma F_x = 0$

$$ku^2 = D \quad \text{(M1)}$$

To get D we will use Power.

Formula for Power:

Power (w) $\smile$ $P = Dv$

Driving force (N) $\smfrown$ velocity (ms⁻¹)

$P = 35v$ ⎤ Substitute:

$D = D$ ⎬

$v = u$ ⎦

$35v = Du$

$D = \dfrac{35v}{u}$

★ We want k.

From (a) we know that:

$\dfrac{100g}{35} = kv^2$, $\therefore k = \dfrac{100g}{35v^2}$ **Sub this for k.**

Substitute D back:

$$ku^2 = \dfrac{35v}{u} \quad \text{(A1)}$$

$$u^2 \times \dfrac{100g}{35v^2} = \dfrac{35v}{u} \quad \text{(M1)}$$

$$\dfrac{100g u^2}{35v^2} = 35\dfrac{v}{u}$$

$$u^3 = \dfrac{1225 v^3}{100g}$$

$\downarrow \sqrt[3]{}$ both sides

$$\boxed{u = 1.08v \text{ to 3sf}} \quad \text{(A1)}$$

u in terms of v

Question 3 continued

Question 3 continued

(Total for Question 3 is 11 marks)

Turn over ▶

4. Two particles, *P* and *Q*, have masses *m* and *em* respectively. The particles are moving on a smooth horizontal plane in the same direction along the same straight line when they collide directly. The coefficient of restitution between *P* and *Q* is *e*, where $0 < e < 1$

Immediately before the collision the speed of *P* is *u* and the speed of *Q* is *eu*.

(a) Show that the speed of *Q* immediately after the collision is *u*.

(6)

(b) Show that the direction of motion of *P* is unchanged by the collision.

(3)

The magnitude of the impulse on *Q* in the collision is $\dfrac{2}{9}mu$

(c) Find the possible values of *e*.

(4)

(a) Diagram

Before | After
u → P (m)   eu → Q (em) | v_P → P (m) v_Q → Q (em)   we want this.
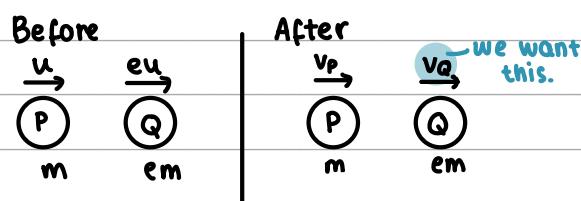
We can use the **conservation of linear momentum** to get an equation.
conservation of linear momentum means: the total momentum before the collision is the **same** as the total momentum after.

Formula: $\quad m_A u_A + m_B u_B = m_A v_A + m_B v_B$
$\qquad\qquad\qquad$ initial velocity $\qquad$ final velocity

Substitute:

$$m(u) + em(eu) = m(v_P) + em(v_Q) \qquad \text{cancel } m's$$

$$u + e^2 u = v_P + e v_Q \qquad \text{Eq.1} \quad \boxed{M1A1}$$

We can use **Newton's Law of Restitution** to get another equation.
Newton's Law of Restitution states that: when two objects collide, their speeds after the collision depend on ① speeds before the collision and ② the material from which they're made.

Formula: $\quad e(u_A - u_B) = v_B - v_A$
$\qquad$ coefficient of restitution $\quad$ initial speed $\quad$ final speed

Substitute:

$$e(u - eu) = v_Q - v_P$$

$$eu - e^2 u = v_Q - v_P \qquad \text{Eq.2} \quad \boxed{M1A1}$$

Solve simultaneously Eq1 and Eq2:

$$\boxed{M1} \quad u + e^2 u = v_P + e v_Q \qquad \text{use elimination method.}$$

$$\underline{\quad eu - e^2 u = -v_P + v_Q \quad} +$$

$$u + eu = (e+1) v_Q \rightarrow \quad v_Q = \frac{u(e+1)}{(e+1)} \quad \therefore \ v_Q = u \quad \text{Shown} \ \boxed{A1}$$

Question 4 continued

(b) Let's get v_P:

$$u + e^2u = v_P + ev_Q \text{ and } v_Q = u$$

$$u + e^2u = v_P + eu \quad \leftarrow \text{Substitute}$$

$$v_P = u - eu + e^2u$$

$$v_P = u(e^2 - e + 1) \quad \boxed{M1}$$

complete square

$$v_P = u\left((e - \tfrac{1}{2})^2 - (\tfrac{1}{2})^2 + 1\right)$$

$$\boxed{A1} \quad v_P = u\left((e - \tfrac{1}{2})^2 + \tfrac{3}{4}\right) \quad \text{completed square form}$$

As $(e - \tfrac{1}{2})^2 > 0$ and $\tfrac{3}{4} > 0$, $(e - \tfrac{1}{2})^2 + \tfrac{3}{4} > 0$.

So, v_P is positive and $\therefore$ v_P does not change direction. $\boxed{A1}$

(c) Use impulse-momentum principle.

Impulse is the Change in momentum

Formula for change in momentum:

$$\Delta \text{momentum} = mv_{\text{final}} - mv_{\text{initial}}$$

mass / velocity

Substitute:

$$I = m(v - u) \quad \text{for particle Q}$$

$$I = em(u - eu) \quad \boxed{M1A1}$$

$$\tfrac{2}{9}mu = em(u - eu)$$

$$\tfrac{2}{9}u = eu(1 - e)$$

$$\tfrac{2}{9} = (e - e^2) \quad \boxed{M1}$$

$$0 = e^2 - e + \tfrac{2}{9}$$

$$0 = 9e^2 - 9e + 2$$

$$0 = (3e - 1)(3e - 2)$$

$$e = \tfrac{1}{3} \qquad e = \tfrac{2}{3}$$

The possible values of e are $e = \tfrac{1}{3}$ and $e = \tfrac{2}{3}$. $\boxed{A1}$

Question 4 continued

(Total for Question 4 is 13 marks)

TOTAL FOR FURTHER MECHANICS 1 IS 40 MARKS